NASA TECHNICAL NOTE

NASA TN D-6452

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NASA TN D-6452

DESIGN OF EXPERIMENTS AS "MULTIPLY TELESCOPING" SEQUENCES OF BLOCKS WITH APPLICATION TO CORROSION BY LIQUID METALS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JULY 1971

1. Report No. NASA TN D-6452	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle DESIGN OF EXPERIMENTS AS "MULTIPLY TELESCOPING" SEQUENCES OF BLOCKS WITH APPLICATION TO CORROSION BY LIQUID METALS		5. Report Date July 1971
		6. Performing Organization Code
7. Author(s) Arthur G. Holms		8. Performing Organization Report No. E-6311
		10. Work Unit No. 129-03
9. Performing Organization Name and Address Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Note
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

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17. Key Words (Suggested by Author(s)) Experimental design; Factorial design; Sequential experiments; Factorial arrangements; Block designs; Response surfaces; Confounding; Aliases; Orthogonality; Fractional replicates; Telescoping designs; Defining contrasts; Steepest descent method		18. Distribution Statement Unclassified - unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 26	22. Price* $3.00

*For sale by the National Technical Information Service, Springfield, Virginia 22151

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SUMMARY

A blocked two-level factorial experiment was designed to measure the elevated-temperature time-dependent corrosion effects of liquid metals on immersed structural materials. Several types of block effects were postulated. The experiment was designed to be "multiply telescoping" so that orthogonal estimates of important parameters could be obtained in the presence of two or more types of block effects, even if the numbers of blocks introducing the several effects were not specified in advance.

The rationale is given for the selection of contrast and treatment generators to provide the options of multiple telescoping. The rationale is also given for identifying parameters aliased together and confounded with blocks, first, if some of the intended options are actually performed and, second, if blocks other than the intended options are actually performed.

INTRODUCTION

Davies and Hay (ref. 5) have noted that experiments could be designed as sequences of orthogonal blocks of two-level fractional-factorial experiments such that observations either from the first block or from a small number of blocks could be used to estimate the coefficients of a simple model. Then, at the option of the experimenter, new blocks of the sequence could be performed such that all acquired observations would be used cumulatively to estimate models of successively greater generality; and the coefficient estimators would be orthogonal to the block effects. General rules for doing this were given by Daniel (ref. 3). A large catalog of the defining contrasts for appropriate plans together with an evaluation of their properties was provided by Addelman (ref. 1). The name "telescoping" has been applied to similar plans in a smaller catalog that also listed the treatments and the aliased parameters (ref. 7).

The foregoing papers assumed that the block effects were the result of a single influence such as an influence correlated with time. The concept of designing for two independent block effects (double confounding) was developed by Holms and Sidik (ref. 8). In that report, rules were given for identifying parameters aliased together and for identifying parameters confounded with block effects in terms of some given group of defining contrasts. Designing for telescoping when more than two independent block effect sources are present is the subject of the present report. Two important aspects of the subject are treated. One is the problem of actually generating appropriate groups of defining contrasts. The other is the problem of identifying parameters aliased together and confounded with block effects, if the blocks actually performed are not those originally contemplated for the telescoping options.

This report discusses multiple telescoping in detail. The discussion will be illustrated by the design of an experiment to observe the elevated temperature corrosion effect of liquid metals on the immersed structural material. Rules are developed for identifying parameters aliased together and for identifying parameters confounded with block effects, if the blocks actually performed differ from the design options.

SYMBOLS

$A, B, \ldots$	defining contrasts
$a, b, \ldots$	treatment has $x_A, x_B, \ldots$ at its high level
$C(i, j, k, l)$	set of defining contrasts
c	number of column blocks
g	number of independent variables (factors)
h	one block contains $(1/2)^h$ of the treatments of a full two-level factorial experiment
I	identity contrast
i, j, k, l	levels of block effects of type i, j, k, l
$x_A, x_B, \ldots$	independent variables
Y	random response variable
y	observed value of Y
$\beta_A, \beta_B, \ldots$	treatment parameters
ϵ	single observation random error

$\mu_i, \mu_j, \ldots$ block effect parameters

(1) treatment with all independent variables at their low level

STRUCTURE OF MULTIPLE TELESCOPING

Treatment Parameters

In brief, the purpose of a two-level fractional-factorial experiment is to estimate the coefficients of an equation. The equation together with assumptions about the experimental error are called the model. The factorial experiment provides estimates of the treatment parameters (coefficients) of an equation of the form

$$Y = \beta_I + \beta_A x_A + \beta_B x_B + \ldots + \beta_g x_g + \beta_{AB} x_A x_B + \beta_{AC} x_{AC} + \ldots + \beta_{g-1,g} x_{g1} x_g$$

$$+ \beta_{ABC} x_A x_B x_C + \ldots + \beta_{g-2,g-1,g} x_{g-2} x_{g-1} x_g + \ldots + \epsilon \qquad (1)$$

where $x_A, \ldots x_g$ are the independent variables, ϵ is the random error, and the β are the treatment parameters.

Notation for Treatments and Defining Contrasts

The independent variables can be standardized (transformed) so that the upper level is represented by $x_A = +1$, $x_B = +1$, and so forth, and the lower level by $x_A = -1$, $x_B = -1$, and so forth. A combination of levels of the independent variables is called a treatment. The notation for the treatments is illustrated by the elements of the rows of table I(a) that are under single letter column headings. The first column of table I(a) gives the familiar notation for treatments described by Davies (ref. 4). The third column of table I(a) and columns to the right give the linear combinations of the observations, which, on division by the number of items in the column, estimate treatment parameters of equation (1) that are subscripted to match the column headings.

The arrangement of table I(a) illustrates the assumption that all of the treatments were performed in a single block. An arrangement illustrating a performance of the same treatments in four blocks is given in table I(b). The resulting confounding of treatment parameters with block effect parameters is illustrated in table II, which will be discussed in the section Illustrative Example with Four Independent Variables.

Additional discussion of notation and terminology was given by Davies (ref. 4) and by Holms and Sidik (ref. 8).

Crossed Classification of Block Effects

Two assumptions are fundamental to planning an experiment for multiple telescoping. One is that the block effects may be classed as crossed. With crossed block effects, each type of block effect can be identified separately from every other type of block effect. Interactions among the block effects can then be defined and estimated. The other assumption is that no block variables interact with treatment variables. If they did, then such block variables would be handled as treatment variables. Such handling would increase the number of parameters to be estimated but would require no basic change in the theory, because crossed classification is assumed for both block and treatment variables.

Maximum Number of Types of Block Effects

The construction of a plan for multiple telescoping begins with a principal block that is a $1/2^h$ replicate of the full 2^g experiment. As such, it contains 2^{g-h} treatment combinations and becomes a full 2^g factorial experiment if expanded by doubling h times. Every such doubling could be accompanied by a new type of block effect, so that as many as h types of block effects can be postulated. The number of independent defining contrasts for the 2^{g-h} replicate is h, so that <u>the maximum number of block effect types is equal to the number of generators of the smallest principal block.</u>

Defining Contrasts

The properties of experiments consisting of the contemplated stopping points of telescoping sequences are mainly determined by the distribution of the word lengths of the defining contrasts of the smallest fraction of the experiment. (Construction of the experiment is simplified by making the smallest fraction a principal block.) Addelman (ref. 1) has tabulated some of the properties that can be achieved. The sequence of expansions he tabulated for a particular family of defining contrasts could constitute one possible path through the many options of multiple telescoping. The other possible paths opened up by the concepts of multiple telescoping deserve investigation.

In many cases, the defining contrasts tabulated by Addelman could serve the purposes of multiple telescoping. In other cases (such as the present corrosion experiment) the smallest block will be smaller than the smallest block tabulated by Addelman, and a larger number of contrast generators will be needed.

Block Parameters

Block effects from sources $i, j, k, \ldots$ can be represented by first degree parameters $\mu_i, \mu_j, \mu_k, \ldots$ and by block interaction parameters $\mu_{ij}, \mu_{ik}, \ldots, \mu_{ijk}, \ldots$ Half replicates of the experiment can then be defined that would not introduce the i-source block effects, or would not introduce the j-source block effects, or would not introduce some other source of block effects. That is to say, if the half replicate were not performed that would, for example, introduce the j-source block effects, then the parameters $\mu_j, \mu_{ij}, \mu_{ijk}$ and all other block parameters having a subscript j would be zero. A particular defining contrast of a half replicate can then be associated with such a particular source of block effects.

GENERATION OF DEFINING CONTRASTS

Corrosion Experiment

The experiment requires the observation of the elevated temperature corrosion effect of a liquid metal on specimen materials. Four specimens can be installed in a furnace, and four furnaces can be installed in a vacuum chamber. The experimenter wanted the option of using one or two vacuum chambers. Because just four specimens can be loaded into a furnace, the basic block size consists of four treatments per block.

The effects of six independent variables are to be observed. The independent variables are time, temperature, oxygen in T-111 structural material, nitrogen in T-111 structural material, presence of TZM in system, and presence of W-25Re in system. The assignment of particular treatments to particular blocks will be discussed in the section DETERMINATION OF TREATMENT GENERATORS. The treatments are assumed to be assigned at random to experimental units within the blocks.

In addition to showing the assignment of particular treatments to particular blocks, the general arrangement of the experiment is shown by table III. The furnaces are identified by the paired numbers (i, j) and these furnace identifications are the row block headings.

The two vacuum chambers are represented by $l = 1$ and $l = 2$. Two successive loadings of each chamber are represented by $k = 1$ and $k = 2$. If only one vacuum chamber were used for all four loadings, then the loadings would be identified by column headings consisting of the paired numbers (k, l). A basic assumption of the present discussion of multiple telescoping is that both the treatment effects and the block effects are crossed classification effects. If two vacuum chambers were used and identified by $l = 1$ and $l = 2$, and if a different set of furnaces were used in the second vacuum chamber, the assumption of crossed block effects would be violated. The furnaces would be nested within vacuum chambers both in the physical sense and also in statistical jargon. The design and analysis of multiply telescoping experiments in the case of a nested classification has not been investigated.

The planned experiment has three independent sources of block effects, namely, (1) differences among the four furnaces, (2) differences between the two vacuum chambers, and (3) differences between the first and the second loading of a vacuum chamber. Because the full replicate consists of four doublings from the smallest block, four independent sources of block effects could have been tolerated (as represented by the four symbols i, j, k, and l).

Defining Contrasts for the Principal Block

A single block of the corrosion experiment can be thought of as a 2^{g-h} experiment where $2^{g-h} = 4$. Then with six independent variables, $g = 6$ and h must be 4; that is, the number of contrast generators for the smallest fractional replicate is 4.

One way of beginning the synthesis of the four needed generators is to use the letters A, B, C, and D singly as generators. The complete group of defining contrasts is then obtained by multiplying these letters together in all possible combinations as shown by table IV. If only the letters A, B, C, and D were used, then the variables x_E and x_F would be omitted from the experiment. These variables are included by attaching combinations of E and F to the generators A, B, C, and D.

An unfortunate constraint on the experiment is that the specimen temperature cannot be varied from specimen to specimen within a furnace. If the temperature is x_A, then x_A is a constant within the block represented by a furnace. Thus, the main effect of x_A must always be confounded with some kind of a block effect, and therefore the single letter A must appear among the defining contrasts of table IV.

In some experimental arrangements, the time that a specimen is exposed to furnace temperature in a vacuum chamber might not be variable from specimen to specimen, in which case some independent variable representing time, say x_B, would also be constant within furnace blocks. Then the single letter B would appear among the defining contrasts of the single block. For the equipment under consideration, specimens can be withdrawn from furnaces without destroying the vacuum, so that the heating time can be varied within furnace blocks.

Thus the letters E and F cannot be suffixed to the letter A of table IV, but these letters can be added to the letters B, C, and D as was done in table IV. The letters E and F then participate in the same multiplication combinations as the A, B, C, and D as shown in table IV. The manner in which the E and F are added to the B, C, and D is arbitrary, except that the distribution of the word lengths might be a consideration. In general, short words are undesirable because they confound low-order treatment parameters with block effects, or else they give poor resolution levels to the fractional replicates.

Experimenters sometimes want the first block to contain a set of control or reference conditions which is assumed to be the treatment combination for which all variables are at their naturally or artificially defined low levels. The block containing such a treatment is called the principal block. The construction of a telescoping sequence is then made particularly convenient if each expansion consists of doubling from the principal block; then the expansions can be achieved by the use of treatment generators that satisfy the rule of even numbers, and the defining contrasts of an expanded stage will be a subgroup of the defining contrasts of the preceding stage. Thus, all stages will consist of principal fractions. So that they will define a principal block, the defining contrasts (table IV) containing an odd number of letters must be given a negative sign.

Defining Contrasts for Additional Blocks and for Block Effects

The experiment beginning with the block of size four can be expanded in a sequence of stages with the important treatment parameter estimates orthogonal to block parameter estimates provided the sequence consists of doublings. With each of the doublings, the defining contrasts will be a subgroup of the defining contrasts before the doubling, and with each of the doublings, an independent source of block effects is assumed to be introduced. Assuming that there are the maximum number of block effect sources (the full replicate consists of four doublings of the principal block) let the sources be represented by the discrete variables i, j, k, l, where the variable takes the value 1 if the doubling has not occurred (block effect has not been introduced) and takes the value 2 if the doubling has occurred (block effect has been introduced). The block effect levels can then be represented by the borderline values of i, j, k, and l of table III.

The subgroups of defining contrasts that will represent successive doublings of the experiment are to be chosen from the 16 defining contrasts of the first block (table IV). Of the possible doublings, the most important is the defining contrast representing doubling with respect to vacuum chamber loadings. This is because the experimenter has said that the probability is high that he will stop the experiment exactly at the half replicate consisting of the four furnaces operated for just two loadings of one vacuum chamber. That replicate is defined by $l = 1$ in table III. So that such an experiment will have the highest possible resolution level (ref. 2), the longest length-defining contrast of table IV (-ABCDE) should define it.

Because there is a small but positive probability of stopping with $k = 1$ and $l = 1$, the contrast defining doubling with respect to k should also be a longer word of table IV. (ABEF was so chosen and its product with -ABCDE, namely, -CDF, should also be a longer word of table IV.)

The experimenter's prior probabilities of some of the block effects were relatively high - 50 percent for a direct effect of differences among furnaces and 20 percent for a difference between vacuum chambers. On the other hand, the experimenter's prior probabilities for stopping at fractional replicates not already discussed were extremely low, so that the matching of other defining contrasts of table IV to the block sources should not be done anticipating such stopping points, but, instead, should be done according to the experimenter's prior probabilities of block effects. The relatively high prior probabilities of furnace block effects suggest that the treatment parameters that must be confounded with the furnace block effects should be treatment parameters presumed to have small prior probabilities of being nonnegligible, namely, the higher order interactions. Thus, the defining contrasts that define expansions of the experiment with respect to i, j, and ij should be some of the longer words of table IV. The words chosen for i and j in table IV must be words such that their product (representing furnace block effect ij) will also be a longer word. The words chosen were -ABC, -BDF, and ACDF.

The four independent defining contrasts just chosen to represent doublings with respect to i, j, k, and l are so identified in table IV. Each such contrast by itself is a defining contrast for a half replicate experiment. Any two of these contrasts provide the generators of a quarter replicate experiment. Thus, many different regular fractions of the experiment from a half replicate on down to a one-eighth replicate may be generated by using combinations of these four generators. In such cases, the defining contrasts so generated will be subgroups of the group generated by all four half replicate generators (as listed for the 1/16 replicate by table IV).

DETERMINATION OF TREATMENT GENERATORS

Rule of Even Numbers

Treatment generators are selected by the rule of even numbers (ref. 4). The selection was made from the display of table V. It lists all the treatments of the full factorial experiment as row headings, and, as column headings, it lists the independent defining contrasts of the half replicate experiments first selected for that purpose in table IV. Also listed in table V are the numbers of letters that occur in common between the treatments and the defining contrasts. According to the rule of even numbers, the treatment generators of the principal block must have only even numbers in common with all of the defining contrast generators. Scanning table V shows that the first two treatments meeting these conditions are the first two treatment generators listed in table VI. Any given treatment generator that doubles the size of the experiment must have an even number of letters in common with all the defining contrast generators, except for the defining contrast generator that disappears on doubling the experiment with the given treatment generator. The first four such treatment generators of table V are indicated by the occurrences of three even numbers underlined together with one odd number not underlined.

Detailed Plan of Experiment

The associations between such treatment generators and the contrast generator they eliminate by the doubling are shown in table VI. The use of the same treatment generators to generate the detailed plan of the experiment is shown by table III.

IDENTIFICATION OF CONFOUNDED PARAMETERS

Illustrative Example with Four Independent Variables

Identification of the treatment parameters that are confounded with block parameters remains to be discussed. The basic method will be developed through a discussion of a hypothetical experiment on four independent variables. (The corrosion experiment with six independent variables will be discussed in the next section.) For the hypothetical experiment on four independent variables, the treatment levels of the full factorial experiment performed as a single block are shown by table I(a). The full factorial experiment of table I(a) was assumed to be subdivided for double telescoping as shown by table I(b).

The groups of defining contrasts for the contemplated replicates will be represented by the symbol $C(i, j)$ where the discrete variable i gives the number of row blocks in the replicate, and the discrete variable j gives the number of column blocks in the replicate. As discussed by Davies (ref. 4) in the chapter on fractional-factorial experiments, the column contrasts of table I(b) that have uniform algebraic signs over the replicates identify the defining contrasts for some of the possible fractional replicates as follows:

$$C(1,1) = I, \ -ABC, \ -ABD, \ CD$$

$$C(1,2) = I, \ -ABD$$

$$C(2,1) = I, \ -ABC$$

$$C(2,2) = I$$

The interrelations of the preceding lists of defining contrasts with possible block parameters will be established by comparing the defining contrasts of table I(b) with postulated block parameters. The postulated block parameters are listed in the first column of table II, namely, (1) a grand mean μ_o, (2) an effect resulting from doubling over rows μ_i, (3) an effect from doubling over columns μ_j, and (4) a row column block interaction effect μ_{ij}.

In the case of just the principal block ($i = 1$ and $j = 1$), table I(b) shows that the contrasts I, -ABC, -ABD, and CD all have the same sign and thus the estimator of the grand mean for this block also estimates a linear combination of treatment parameters with these contrasts as subscripts. The treatment parameters are all aliased together and confounded with the grand mean as indicated under $C(1, 1)$ of table II.

If the experiment is now doubled by adding the block for which $j = 2$, then the contrast under ABD (table I(b)) is of uniform sign so that β_{ABD} is confounded with the grand mean β_I, as represented by row μ_o and column $C(1, 2)$ of table II. Furthermore, the contrast ABC has opposite signs for the two blocks, so that the estimate of the parameter β_{ABC} is also an estimate of the difference between the blocks. The same statement can be made with respect to the CD contrast of table I(b) so that, as represented in table II, β_{CD} and β_{ABC} are both confounded with μ_j.

Similar statements can be made about the aliased and confounded parameters resulting from the $C(2, 1)$ replicate.

In the case of the full replicate, the column under I of table I(b) provides the estimate of the grand mean, and that parameter can be called β_I or μ_o. The contrast under ABD has a change of sign from $i = 1$ to $i = 2$ and thus provides an estimate of the treatment parameter β_{ABD} confounded with the block parameter μ_i. The contrast under ABC changes sign when j changes from $j = 1$ to $j = 2$ and thus provides an estimate of the confounded combination of μ_j and β_{ABC}. The contrast under CD has positive signs for both $i = 1$, $j = 1$ and $i = 2$, $j = 2$, but it has negative signs for both $i = 1$, $j = 2$ and $i = 2$, $j = 1$. Thus, the contrast CD estimates the treatment parameter β_{CD} confounded with the block interaction parameter μ_{ij}. In summary, the first two columns of table II show which of the defining contrasts of the smallest fractional replicate become estimators of the postulated block effects in the full replicate.

Table II also shows how the treatment parameters confounded with block parameters can be quickly identified for all the contemplated fractional replicates. The defining contrasts for the contemplated fractional replicates are given by the subscripts of the treatment parameters aliased with μ_o (listed in the μ_o row of table II). Then, for a block effect that can exist, the defining contrast of the full replicate for that block parameter (in the second column of table II) multiplies the defining contrasts of the particular fractional replicate (subscripts in the μ_o row) to give the subscripts of the treatment parameters confounded with the particular block parameter. For example, the C(1, 2) replicate has defining contrasts I and -ABD as shown by the subscripts of the treatment parameters in the μ_o row. Doubling the C(1, 1) replicate to the C(1, 2) replicate is done with respect to the j source of block effects so that the μ_j parameter is postulated. In the full replicate (as shown by the second column of table II), the μ_j parameter has defining contrast -ABC. The products of -ABC with the defining contrasts of the C(1, 2) replicate (namely, I and -ABD) are -ABC and CD, and thus the treatment parameters confounded with μ_j are as listed under C(1, 2) of table II, namely, $-\beta_{ABC} + \beta_{CD}$.

Identification of Confounded Parameters in the Corrosion Experiment

Identification of the estimators confounded with block effects is to be done by identifying the treatment parameters that are confounded with block parameters. This is to be done through the groups of defining contrasts in the manner just described for the illustrative example with four independent variables. As a start, block parameters are postulated to represent the physical plan of the experiment.

Individual furnaces might degrade with time, either gradually or suddenly. Therefore, all conceivable interaction effects might exist between the furnace blocks and the time blocks. Thus besides the furnace effects, μ_i, μ_j, μ_{ij}, and the time effects, μ_k, μ_l, μ_{kl}, the furnace time interactions, μ_{ik}, μ_{il}, μ_{jk}, μ_{jl}, μ_{ijk}, μ_{ijl}, μ_{ikl}, μ_{jkl}, μ_{ijkl} are postulated.

A notation is needed for the groups of defining contrasts that define the contemplated stopping points. The symbol $C(i, j, k, l)$ will represent such groups for the contemplated replicates. For any such fractional replicate, those variables among i, j, k, l that are identified with a generator of the given replicate will remain at their low level, whereas doubling with respect to a source of block effects will be represented by the presence of the variable at its high level. The association between the variables representing sources of block effects and the generators of fractional replicate defining contrasts was discussed in connection with table VI.

The scheme for identifying the aliased combinations of model parameters that are confounded with block parameters is basically the same as that displayed for the hypothetical experiment on four independent variables by table II.

The scheme for the corrosion experiment is displayed by table VII. The first column lists the block parameters and the second column gives the experimentor's prior probabilities that such block effects exist. The four defining contrasts shown in table VI for expansions with respect to i, j, k, and l are listed in the third column of table VII. These four defining contrasts are then multiplied together as required by the multiplications of i, j, k, and l in the first column. The resulting products are the contrasts that define the block effects that were listed in the first column. As would be expected, these contrasts also occur in the first column of table IV.

The defining contrasts that estimate the block effects in the full replicate are given by the third column of table VII. These contrasts are also the subscripts of the treatment parameters estimated by the contrasts; therefore, such treatment parameters are confounded with the associated block parameters in the full replicate.

The defining contrasts for the larger fractional replicates as generated from table VI are given in the fourth through 13th columns in the μ_0 row of table VII. Under these replicates, the block effects of the full replicate must disappear according to the fraction of the full replicate that is not performed. For the fraction that is performed, the block effect contrast of the full replicate experiment multiplies the fractional replicate defining contrasts to give the subscripts of the treatment parameters that are aliased together and confounded with the block effect parameters (as listed in the columns of table VII).

An alternative to the identification procedure illustrated by this discussion of table VII was given for double telescoping by Holms and Sidik (ref. 8). That procedure is equally applicable to multiple telescoping.

MISSING BLOCKS

Contemplated Blocks

Thus far, the blocks actually performed were assumed to be those that conform to the options designated by lower or upper levels of i, j, k, and l as in table III. Such options are compatible with the defining contrasts given in tables IV and VII. The defining contrasts are assumed to have been selected to be optimal from some point of view, and these contrasts determine the structure of the confounding between the model parameters and the block parameters as illustrated in table VII. Remaining to be discussed is the subject of the estimation of parameters when the blocks performed do not conform to any of the options implied by the $C(i,j,k,l)$ notation of table VII.

For example, if at any time during the experimenting, one or more of the furnaces were found to be defective, then such missing data would be equivalent to a limitation on the previously listed values of i and j. According to what particular furnaces were defective, the attribute of orthogonality could be achieved by suitably restricting the values of i or j or both. (Such a restriction might discard some data beyond the data that were missing.)

Regression Methods

In any case, the methods of regression analysis (Draper and Smith (ref. 6)) are available as a back stop analysis for an experiment that has not been completed according to plan. When it must be used, the success of such a salvage operation will depend (1) on what has been removed from the plan, (2) on how well the statistician knows the true conditions of the experiment, and (3) on how intelligently he matches the form of the regression equation to his knowledge of the true conditions of the experiment.

Orthogonal Blocks

The salvage operation of regression or more sophisticated analysis need not be used even if the blocks actually performed differ from those contemplated in the telescoping options, provided that the performed blocks constitute a regular fractional replicate. In such cases, an effort beyond that already described will be needed to construct the model that can be estimated. An example will be used to illustrate some techniques that are useful in matching the estimates to the aliased combinations of parameters.

Suppose that accidents to the equipment were such that the treatments performed are only those of furnace numbers 2 and 3 and vacuum chamber loadings 1 and 4. These treatments are shown in table III by the following combinations of i, j, k, and l:

i, j, k, l	Treatments
2, 1, 1, 1	abcd, ae, adf, abcef
1, 2, 1, 1	ab, acde, acf, abdef
2, 1, 2, 2	a, abcde, abcf, adef
1, 2, 2, 2	acd, abe, abdf, acef

Identification of Defining Contrasts

The problem of putting the treatments and their associated observations in standard order for the purpose of using Yates' algorithm will be approached by ignoring certain treatment letters, somewhat in the manner of Daniel (ref. 3). The procedure is started by putting the treatments in what would be standard order for the full replicate. This has been done in table VIII. Furthermore, the 16 treatments would constitute a full factorial experiment if only four independent variables were present, and, therefore, two of the six variables actually present must be ignored. The ignored variables are enclosed in parentheses in table VIII. They were selected so that the letters not in parentheses would form a standard order pattern. The letters in standard order pattern are b, d, e, and f.

Yates' method with a full factorial experiment on four independent variables x_B, x_D, x_E, and x_F then provides estimates in the row-wise order of the treatment parameters occurring in first position among the alias sets of table VIII. (For example, the first four estimates are estimates of β_I, β_B, β_D, and β_{BD}.)

The alias sets remain to be determined and this will be done in terms of defining contrasts. The defining contrasts are to be determined making particular use of the treatment letters that were enclosed in parentheses in table VIII. Inspection of the treatment column of table VIII shows that the letter a is present everywhere, thus the contrast A is equal to the contrast I; that is,

$$A = I$$

Furthermore, the letter c is present whenever there is an odd number of letters d or f, independently of e. Thus, x_C takes on its positive value whenever just one of x_D or x_F takes on its positive value, the other being negative. Under this circumstance, the governing relation is

$$C = -DF$$

or

$$I = -CDF$$

The complete group of defining contrasts is, therefore,

$$[I, A, -CDF, -ACDF]$$

The preceding group is now used to determine the last three of each of the alias sets of four treatment parameters of table VIII. This is done by multiplying the subscripts of the first members (which have already been determined) in the usual manner with the preceding group of defining contrasts. The results are listed in table VIII.

Nelder (ref. 9) has also discussed the identification of aliased treatment parameter combinations with estimates from Yates' algorithm, when a fraction other than the principal fraction has been used for the fractional replicate.

Identification of Block Parameters

The confoundings among block parameters and treatment parameters remain to be identified. For this example of missing blocks, the contrast groups will be labeled by the symbol $C(r, c)$, which is defined in table IX.

For $C(2, 1)$ the treatments may be written in the standard order as

(a)	(b)	(c)	(d)		
(a)				e	
(a)			(d)		f
(a)	(b)	(c)		e	f

where the letters not in parentheses are in the standard order of a two-level full-factorial experiment. The letters in parentheses show that the high levels of these treatments are matched to the positive and negative values of x_E and x_F according to the rules

$$A = I$$

$$B = EF$$

$$C = EF$$

$$D = -E$$

and the defining contrast generators are, therefore, A, BEF, CEF, and -DE. The preceding list of generators permits construction of the complete group of defining contrasts as listed for C(2, 1) in table IX.

For expansion from $r = 2$ to $r = 3$ with $c = 1$, table III shows that the treatment blocks are

abcd
ae
adf
abcef

ab
acde
acf
abdef

Arranged in standard order these treatments are

(a)	(b)				
(a)	(b)	(c)	d		
(a)				e	
(a)		(c)	d	e	
(a)		(c)			f
(a)			d		f
(a)	(b)	(c)		e	f
(a)	(b)		d	e	f

The letters in a Yates' order pattern are the letters that are not in parentheses, and they constitute the treatments of a full 2^3 experiment. The letters in parentheses are seen to have their levels subject to the rules

$$A = I$$

$$B = EF$$

$$C = -DF$$

so that the defining contrast generators are A, BEF, -CDF. The complete group of defining contrasts is given in the C(3, 1) column of table IX.

For the expansion from $c = 1$ to $c = 4$ with $r = 2$ the treatment blocks are

abcd
ae
adf
abcef

a
abcde
abcf
adef

Arranged in standard order these treatments are

(a)					
(a)	(b)	(c)	d		
(a)				e	
(a)	(b)	(c)	d	e	
(a)	(b)	(c)			f
(a)			d		f
(a)	(b)	(c)		e	f
(a)			d	e	f

The letters in parentheses are seen to have their levels subject to the rules

$$A = I$$

$$B = -DF$$

$$C = -DF$$

so that the defining contrast generators are A, -BDF, and -CDF. The complete group of defining contrasts is given in the C(2, 4) column of table IX.

Identification of treatment parameters confounded with block effects can now be done according to rules given by Holms and Sidik (ref. 8). Thus, from the C(3, 4) column of table IX the treatment parameters confounded with the grand mean are β_I, β_A, $-\beta_{CDF}$, and $-\beta_{ACDF}$ as listed in table VIII. The contrasts in C(2, 4) that are not in C(3, 4) identify the parameters of C(3, 4) confounded with the row doubling as β_{BC}, $-\beta_{BDF}$, β_{ABC}, and $-\beta_{ABDF}$. These parameters are identified by the superscript r in table VIII. The contrasts in C(3, 1) that are not in C(3, 4) identify the parameters of C(3, 4) confounded with the column doubling as β_{BEF}, β_{ABEF}, $-\beta_{BCDE}$, and $-\beta_{ABCDE}$. These parameters are identified by the superscript c in table VIII. The contrasts in C(2, 1) that are not in C(3, 1) and not in C(2, 4) of table IX identify the model parameters confounded with row-column interaction block effects as β_{CEF}, $-\beta_{DE}$, β_{ACEF}, and $-\beta_{ADE}$. These parameters are identified by the superscript rc in table VIII.

CONCLUDING REMARKS

A blocked two-level factorial experiment was designed to measure the elevated-temperature time-dependent corrosion effects of a liquid metal on some immersed structural materials. Several types of block effects were postulated. The experiment was designed to be multiply telescoping so that orthogonal estimates of important treatment parameters could be obtained, even if the numbers of blocks introducing the several effects were not specified in advance.

Subjects receiving particular attention were

1. The construction of the complete group of defining contrasts
2. The matching of particular defining contrasts to particular options of fractional replication
3. The matching of particular defining contrasts to particular block effects
4. The selection of treatment generators to provide the options of multiple telescoping

5. The identification of aliased and confounded parameters
6. The consequences resulting from failure to perform exactly those blocks specified by the options

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 25, 1971,
129-03.

REFERENCES

1. Addelman, Sidney: Sequences of Two-Level Fractional Factorial Plans. Technometrics, vol. 11, no. 3, Aug. 1969, pp. 477-509.
2. Box, G. E. P.; and Hunter, J. S.: The 2^{k-p} Fractional Factorial Designs. I. Technometrics, vol. 3, no. 3, Aug. 1961, pp. 311-351.
3. Daniel, Cuthbert: Fractional Replication in Industrial Research. Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability. Vol. 5. University of California Press, 1956, pp. 87-98.
4. Davies, Owen L., ed.: The Design and Analysis of Industrial Experiments. Second ed., Hafner Publishing Co., 1960.
5. Davies, O. L.; and Hay, W. A.: The Construction and Uses of Fractional Factorial Designs in Industrial Research. Biometrics, vol. 6, 1950, pp. 233-249.
6. Draper, N. R.; and Smith, H.: Applied Regression Analysis. John Wiley & Sons, Inc., 1966.
7. Holms, Arthur G.: Designs of Experiments as Telescoping Sequences of Blocks for Optimum Seeking (As Intended for Alloy Development). NASA TN D-4100, 1967.
8. Holms, Arthur G.; and Sidik, Steven M.: Design of Experiments as "Doubly Telescoping" Sequences of Blocks with Application to a Nuclear Reactor Experiment. NASA TN D-5369, 1969.
9. Nelder, J. A.: Identification of Contrasts in Fractional Replicates of 2^n Experiments. Appl. Statistics, vol. 12, 1963, pp. 38-43.

TABLE I. - 2^4 EXPERIMENT

(a) Full replicate in one block

Treatment	Response	Matrix of independent variables															
		I	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD
(1)	y_1	+1	-1	-1	+1	-1	+1	+1	-1	-1	+1	+1	-1	+1	-1	-1	+1
a	y_2	+1	+1	-1	-1	-1	-1	+1	+1	-1	-1	+1	+1	+1	+1	-1	-1
b	y_3	+1	-1	+1	-1	-1	+1	-1	+1	-1	+1	-1	+1	+1	-1	+1	-1
ab	y_4	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1	+1	+1	+1	+1
c	y_5	+1	-1	-1	+1	+1	-1	-1	+1	-1	+1	+1	-1	-1	+1	+1	-1
ac	y_6	+1	+1	-1	-1	+1	+1	-1	-1	-1	-1	+1	+1	-1	-1	+1	+1
bc	y_7	+1	-1	+1	-1	+1	-1	+1	-1	-1	+1	-1	+1	-1	+1	-1	+1
abc	y_8	+1	+1	+1	+1	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1
d	y_9	+1	-1	-1	+1	-1	+1	+1	-1	+1	-1	-1	+1	-1	+1	+1	-1
ad	y_{10}	+1	+1	-1	-1	-1	-1	+1	+1	+1	+1	-1	-1	-1	-1	+1	+1
bd	y_{11}	+1	-1	+1	-1	-1	+1	-1	+1	+1	-1	+1	-1	-1	+1	-1	+1
abd	y_{12}	+1	+1	+1	+1	-1	-1	-1	-1	+1	+1	+1	+1	-1	-1	-1	-1
cd	y_{13}	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1
acd	y_{14}	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1
bcd	y_{15}	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1
abcd	y_{16}	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1

(b) With double telescoping among four blocks

Row i	Column j	Treatment	Matrix of independent variables															
			I	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD
1	1	(1)	+1	-1	-1	+1	-1	+1	+1	-1	-1	+1	+1	-1	+1	-1	-1	+1
		acd	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1
		bcd	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1
		ab	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1	+1	+1	+1	+1
1	2	ad	+1	+1	-1	-1	-1	-1	+1	+1	+1	+1	-1	-1	-1	-1	+1	+1
		c	+1	-1	-1	+1	+1	-1	-1	+1	-1	+1	+1	-1	-1	+1	+1	-1
		abc	+1	+1	+1	+1	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1
		bd	+1	-1	+1	-1	-1	+1	-1	+1	+1	-1	+1	-1	-1	+1	-1	+1
2	1	d	+1	-1	-1	+1	-1	+1	+1	-1	+1	-1	-1	+1	-1	+1	+1	-1
		ac	+1	+1	-1	-1	+1	+1	-1	-1	-1	-1	+1	+1	-1	-1	+1	+1
		bc	+1	-1	+1	-1	+1	-1	+1	-1	-1	+1	-1	+1	-1	+1	-1	+1
		abd	+1	+1	+1	+1	-1	-1	-1	-1	+1	+1	+1	+1	-1	-1	-1	-1
2	2	a	+1	+1	-1	-1	-1	-1	+1	+1	-1	-1	+1	+1	+1	+1	-1	-1
		cd	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1
		abcd	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
		b	+1	-1	+1	-1	-1	+1	-1	+1	-1	+1	-1	+1	+1	-1	+1	-1

TABLE II. - TREATMENT PARAMETERS CONFOUNDED

WITH BLOCK PARAMETERS

Block effects		Replicate,[a] C(i, j)			
Parameter	Defining contrast	C(2, 2)	C(2, 1)	C(1, 2)	C(1, 1)
		Treatment parameters			
μ_o	I	β_I	$\beta_I-\beta_{ABC}$	$\beta_I-\beta_{ABD}$	$\beta_I-\beta_{ABD}-\beta_{ABC}+\beta_{CD}$
μ_i	-ABD	$-\beta_{ABD}$	$-\beta_{ABD}+\beta_{CD}$		
μ_j	-ABC	$-\beta_{ABC}$		$-\beta_{ABC}+\beta_{CD}$	
μ_{ij}	CD	β_{CD}			

[a]Obtained by doubling to level i with respect to the i source of block effects and by doubling to level j with respect to the j-source of block effects.

TABLE III. - PLAN OF EXPERIMENT

Block effect source		Furnace number, r	Vacuum chamber, l			
			1		2	
			Loading, k			
			1	2	1	2
			Loading number, c			
j	i		1	2	3	4
1	1	1	(1) bcde bcf def	ac abde abf acdef	abd ace acdf abef	bcd e df bcef
	2	2	abcd ae adf abcef	bd ce cdf bef	c bde bf cdef	a abcde abcf adef
2	1	3	ab acde acf abdef	bc de f bcdef	d bce bcdf ef	acd abe abdf acef
	2	4	cd be bdf cef	ad abce abcdf aef	abc ade af abcdef	b cde cf bdef

TABLE IV. - DEFINING CONTRASTS OF SMALLEST BLOCK C(1, 1, 1, 1)

Defining contrast	Block effect source
I	
A	
-B EF	
-C EF	
D E	
AB EF	k
AC EF	
-AD E	
BC	
-BD F	j
-CD F	
-ABC	i
ABD F	
ACD F	
BCD E	
-ABCD E	l

TABLE V. - SELECTION OF TREATMENT GENERATORS

Treatment	Independent defining contrasts				Treatment	Independent defining contrasts			
	-ABC	-BDF	ABEF	-ABCDE		-ABC	-BDF	ABEF	-ABCDE
(1)	0	0	0	0	f	0	1	1	0
a	1	0	1	1	af	1	1	2	1
b	1	1	1	1	bf	1	2	2	1
ab	2	1	2	2	abf	2	2	3	2
c	1	0	0	1	cf	1	1	1	1
ac	2	0	1	2	acf	2	1	2	2
bc	2	1	1	2	bcf	2	2	2	2
abc	3	1	2	3	abcf	3	2	3	3
d	0	1	0	1	df	0	2	1	1
ad	1	1	1	2	adf	1	2	2	2
bd	1	2	1	2	bdf	1	3	2	2
abd	2	2	2	3	abdf	2	3	3	3
cd	1	1	0	2	cdf	1	2	1	2
acd	2	1	1	3	acdf	2	2	2	3
bcd	2	2	1	3	bcdf	2	3	2	3
abcd	3	2	2	4	abcdf	3	3	3	4
e	0	0	1	1	ef	0	1	2	1
ae	1	0	2	2	aef	1	1	3	2
be	1	1	2	2	bef	1	2	3	2
abe	2	1	3	3	abef	2	2	4	3
ce	1	0	1	2	cef	1	1	2	2
ace	2	0	2	3	acef	2	1	3	3
bce	2	1	2	3	bcef	2	2	3	3
abce	3	1	3	4	abcef	3	2	4	4
de	0	1	1	2	def	0	2	2	2
ade	1	1	2	3	adef	1	2	3	3
bde	1	2	2	3	bdef	1	3	3	3
abde	2	2	3	4	abdef	2	3	4	4
cde	1	1	1	3	cdef	1	2	2	3
acde	2	1	2	4	acdef	2	2	3	4
bcde	2	2	2	4	bcdef	2	3	3	4
abcde	3	2	3	5	abcdef	3	3	4	5

TABLE VI. - ASSOCIATION OF TREATMENT AND CONTRAST

GENERATORS WITH BLOCK EFFECT SOURCES

Treatment generator	Block effect source introduced with doubling by treatment generator	Defining contrast eliminated by doubling with treatment generator
bcde		
bcf		
abcd	i	-ABC
ab	j	-BDF
ac	k	ABEF
abd	l	-ABCDE

TABLE VII. - TREATMENT PARAMETERS CONFOUNDED WITH BLOCK PARAMETERS

Block effects			Fractional replicate,[a] C(i, j, k, ℓ)									
Parameter	Prior probability	Defining contrasts	C(1, 2, 2, 2)	C(2, 1, 2, 2)	C(2, 2, 1, 2)	C(2, 2, 2, 1)	C(1, 1, 2, 2)	C(1, 2, 1, 2)	C(1, 2, 2, 1)	C(2, 1, 1, 2)	C(2, 1, 2, 1)	C(2, 2, 1, 1)
			Stopping probability									
			0.02	0.02	0.02	0.34	0.01	0.01	0.01	0.01	0.01	0.05
			Subscripts of treatment parameters									
μ_{o}	1.00	I	I -ABC	I -BDF	I ABEF	I -ABCDE	I -ABC -BDF ACDF	I -ABC ABEF -CEF	I -ABC -ABCDE DE	I -BDF ABEF -ADE	I -BDF -ABCDE ACEF	I ABEF -ABCDE -CDF
μ_{i}	0.50	-ABC		-ABC ADCF	-ABC -CEF	-ABC DE				-ABC ACDF -CEF BCDE	-ABC ACDF DE -BEF	-ABC -CEF DE ABDF
μ_{j}	0.50	-BDF	-BDF ACDF		-BDF -ADE	-BDF ACEF		-BDF ACDF -ADE BCDE	-BDF ACDF ACEF -BEF			-BDF -ADE ACEF BC
μ_{k}	0.10	ABEF	ABEF -CEF	ABEF -ADE		ABEF -CDF	ABEF -CEF -ADE BCDE		ABEF -CEF -CDF ABDF		ABEF -ADE -CDF BC	
μ_{ℓ}	0.20	-ABCDE	-ABCDE DE	-ABCDE ACEF	-ABCDE -CDF		-ABCDE DE ACEF -BEF	-ABCDE DE -CDF ABDF		-ABCDE ACEF -CDF BC		
μ_{ij}	0.50	ACDF			ACDF BCDE	ACDF -BEF						ACDF BCDE -BEF -A
μ_{ik}	0.10	-CEF		-CEF BCDE		-CEF ABDF					-CEF BCDE ABDF -A	
$\mu_{i\ell}$	0.10	DE		DE -BEF	DE ABDF					DE -BEF ABDF -A		
μ_{jk}	0.10	-ADE	-ADE BCDE			-ADE BC			-ADE BCDE BC -A			
$\mu_{j\ell}$	0.10	ACEF	ACEF -BEF		ACEF BC			ACEF -BEF BC -A				
$\mu_{k\ell}$	0.10	-CDF	-CDF ABDF	-CDF BC			-CDF ABDF BC -A					
μ_{ijk}	0.10	BCDE				BCDE -A						
$\mu_{ij\ell}$	0.10	-BEF			-BEF -A							
$\mu_{ik\ell}$	0.10	ABDF		ABDF -A								
$\mu_{jk\ell}$	0.10	BC	BC -A									
$\mu_{ijk\ell}$	0.10	-A										

[a]C(i, j, k, ℓ) is the fractional replicate which, if doubled with respect to i, j, k, or ℓ, contains i, j, k, or ℓ type block effect sources.

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TABLE VIII. - TREATMENTS AND ESTIMATES FOR

EXAMPLE OF MISSING BLOCKS

Treatments	Estimates[a]			
(a)	β_I	$+\beta_A$	$-\beta_{CDF}$	$-\beta_{ACDF}$
(a)b	β_B	$+\beta_{AB}$	$-\beta_{BCDF}$	$-\beta_{ABCDF}$
(a)(c)d	β_D	$+\beta_{AD}$	$-\beta_{CF}$	$-\beta_{ACF}$
(a)b(c)d	β_{BD}	$+\beta_{ABD}$	$-\beta_{BCF}$	$-\beta_{ABCF}$
(a)e	β_E	$+\beta_{AE}$	$-\beta_{CDEF}$	$-\beta_{ACDEF}$
(a)be	β_{BE}	$+\beta_{ABE}$	$-\beta_{BCDEF}$	$-\beta_{ABCDEF}$
(a)(c)de	$(^a\beta_{DE}$	$+\beta_{ADE}$	$-\beta_{CEF}$	$-\beta_{ACEF})^{rc}$
(a)b(c)de	β_{BDE}	$+\beta_{ABDE}$	$-\beta_{BCEF}$	$-\beta_{ABCEF}$
(a)(c)f	β_F	$+\beta_{AF}$	$-\beta_{CD}$	$-\beta_{ACD}$
(a)b(c)f	β_{BF}	$+\beta_{ABF}$	$-\beta_{BCD}$	$-\beta_{ABCD}$
(a)df	β_{DF}	$+\beta_{ADF}$	$-\beta_C$	$-\beta_{AC}$
(a)bdf	$(\beta_{BDF}$	$+\beta_{ABDF}$	$-\beta_{BC}$	$-\beta_{ABC})^r$
(a)(c)ef	β_{EF}	$+\beta_{AEF}$	$-\beta_{CDE}$	$-\beta_{ACDE}$
(a)b(c)ef	$(\beta_{BEF}$	$+\beta_{ABEF}$	$-\beta_{BCDE}$	$-\beta_{ABCDE})^c$
(a)def	β_{DEF}	$+\beta_{ADEF}$	$-\beta_{CE}$	$-\beta_{ACE}$
(a)bdef	β_{BDEF}	$+\beta_{ABDEF}$	$-\beta_{BCE}$	$-\beta_{ABCE}$

[a]rc denotes estimates confounded with row-column interaction effects; r with row block effects; and c with column block effects.

TABLE IX. - DEFINING CONTRASTS

WITH MISSING BLOCKS

[The symbol C(r, c) means the fractional replicate with row and column blocks of table III as follows: C(2, 1) indicates second row, first column; C(3, 1) second and third rows, first column; C(2, 4) second row, first and fourth columns; C(3, 4) second and third rows, first and fourth columns.]

Fractional replicate, C(r, c)			
C(2, 1)	C(3, 1)	C(2, 4)	C(3, 4)
Defining contrasts			
I	I	I	I
A	A	A	A
BEF	BEF		
CEF			
-DE			
ABEF	ABEF		
ACEF			
-ADE			
BC		BC	
-BDF		-BDF	
-CDF	-CDF	-CDF	-CDF
ABC		ABC	
-ABDF		-ABDF	
-ACDF	-ACDF	-ACDF	-ACDF
-BCDE	-BCDE		
-ABCDE	-ABCDE		

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